HEAT AND MASS TRANSFER IN A TURBULENT NONEQUILIBRIUM FLOW IN AN ANNULAR CHANNEL. PART 2. THERMAL INITIAL SEGMENT

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Theoretical expressions are derived for the heat and mass transfer characteristics in a turbulent reacting flow in the initial themal section of a ring channel. The corresponding Sturm–Liouville problem has been solved numerically. Eigenvalues, eigenfuctions, and expansion coefficients have been tabulated. Quantitative criteria are derived for the asymptotic cases of quasifrozen and quasiequilibrium flow states, together with the expression for the length of the initial thermal section in a chemically nonequilibrium turbulent flow.

We consider a hydrodynamically stabilized chemically nonequilibrium flow in a channel formed by coaxial cylinders having inert impermeable surfaces, where there is a constant heat flux (q_{c1}=const, q_{c2}=0) .

There is homogeneous reversible reaction involving the dissociation of nitrogen dioxide; the inlet flow has a uniform profile for the temperature T_0 and a chemically equilibrium composition $x_0=x_e(T_0)$.

The differential equations and simplifying assumptions have been given in [1]; to derive the theoretical heat and mass transfer characteristics, one needs to solve

$$f(\xi)F(\xi)G^2\frac{\partial P}{\partial \eta}=\frac{\partial}{\partial \xi}\left[F(\xi)\frac{\partial P}{\partial \xi}\right]-\gamma_T^2 PF(\xi),$$
$$P(\xi,\ 0)=0,\quad \left(\frac{\partial P}{\partial \xi}\right)_{\xi=0}=-1,\quad \left(\frac{\partial P}{\partial \xi}\right)_{\xi=1}=0, \tag{1}$$

in which $P=W/\varkappa G$, $W=(1+\varkappa)\Theta-\Theta_f$.

Gretz's method [1] has been applied to (1): separating the solution into a part with stabilized heat transfer $P_\infty(\xi)$ and an intial thermal section $P_0(\xi, \eta)$:

$$P(\xi,\ \eta)=P_\infty(\xi)+P_0(\xi,\ \eta),$$
$$P_\infty(\xi)=\lim_{\eta\to\infty}P(\xi,\ \eta),\quad \lim_{\eta\to 0}P_0(\xi,\ \eta)=0. \tag{2}$$

For the part with stabilized heat transfer, we have a boundary-value problem of the second kind:

$$(F(\xi)P_\infty')'-\gamma_T^2 P_\infty F(\xi)=0,$$
$$P_\infty'(0)=-1,\quad P_\infty'(1)=0. \tag{3}$$

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The (3) treatment has been solved by the pivot method [1].

The heat and mass transfer in the initial thermal section can be described by

$$G^2F(\xi)f(\xi)\frac{\partial P_0}{\partial \eta}=\frac{\partial}{\partial \xi}\left[F(\xi)\frac{\partial P_0}{\partial \xi}\right]-\gamma_T^2P_0F(\xi),$$
$$P_0(\xi,\ 0)=-P_\infty(\xi),\quad \left(\frac{\partial P_0}{\partial \xi}\right)_{\xi=0}=0,\quad \left(\frac{\partial P_0}{\partial \xi}\right)_{\xi=1}=0. \tag{4}$$

We represent the solution to (4) as an eigenfunction series:

$$P_0=\sum_{n=0}^{\infty}C_n\psi_n(\xi)\exp(-\varepsilon_n^2\eta),\quad C_n=\frac{-\int_0^1 P_\infty\psi_nFfd\xi}{\int_0^1 Ff\psi_n^2d\xi}. \tag{5}$$

We substitute (5) into (4) to get an eigenvalue problem:

$$(F\psi_n')'+\lambda_n^2Ff\psi_n-\gamma_T^2\psi_nF=0,$$
$$\psi_n'(0)=0,\quad \psi_n'(1)=0, \tag{6}$$

in which $\lambda_n^2=(\varepsilon_nG)^2$ is a reduced eigenvalue.

The form of (6) is identical with that in case we have solved previously [2] for the eigenvalues for a tube (the geometry is incorporated by means of the integral coordinate ξ, the functions $F(\xi)$ and $f(\xi)$, and the thermal parameter γ_T^2 for the chemical disequilibrium in turbulent flow).

By definition

$$\bar{\psi}_n=\frac{\int_0^1\psi_nFfd\xi}{\int_0^1 Ffd\xi}=\frac{2G(1-k)}{1+k}\int_0^1\psi_nFfd\xi. \tag{7}$$

We integrate (6) from 0 to 1 and use the boundary conditions to get

$$\frac{2G(1-k)}{1+k}\lambda_n^2\int_0^1\psi_nFfd\xi=\gamma_T^2\frac{2G(1-k)}{1+k}\int_0^1\psi_nFd\xi. \tag{8}$$

Substitution of (8) into (7) gives

$$\bar{\psi}_n=\frac{\gamma_T^2 2G(1-k)}{\lambda_n^2(1+k)}\int_0^1\psi_nFd\xi. \tag{9}$$

The operator $L\psi_n=(F\psi_n')'-\gamma_T^2F\psi_n$ is self-conjugate because ψ_n satisfies homogeneous conditions at both boundaries; the self-conjugate feature of $L\psi_n$ implies that the eigenfunctions are orthogonal with weights Fe [3]:

$$\int_0^1\psi_n\psi_mFfd\xi=0. \tag{10}$$

To eliminate the possibility that roots might be overlooked in the numerical solution, we made a priori estimates for the λ_n^2; the [2] method gave approximate theoretical expressions for the eigenvalues:

TABLE 1. Eigenvalues (Re = 10^5; k = 0.5; Pr_f = 0.7; Pr_T = 0.8; G = 0.1384; f_m = 1.143; g_m = 68.55)

n	a_n	λ^2_{nf} nu	$\gamma = 10$ ($\gamma_T^2 = 1{,}917$)			$\gamma = 10^2$ ($\gamma_T^2 = 191{,}7$)			$\gamma = 10^3$ ($\gamma_T^2 = 1{,}917 \cdot 10^4$)		
			λ^2_n th	λ^2_n nu	ε^2_n nu $\cdot 10^{-3}$	λ^2_n th	λ^2_n nu	ε^2_n th $\cdot 10^{-2}$	λ^2_n th $\cdot 10^{-4}$	λ^2_n nu $\cdot 10^{-4}$	ε^2_n th $\cdot 10^{-5}$
0	0	0	1,917	1,890	0,9846	191,7	181,4	94,63	1,917	1,688	8,809
1	3,832	15,93	18,79	17,96	9,37	208,4	212,7	111,0	1,918	1,720	8,975
2	7,016	60,75	62,67	62,77	32,75	248,1	262,1	136,7	1,922	1,752	9,141
3	10,173	125,0	120,7	127,1	66,31	310,4	329,7	172,0	1,928	1,785	9,311
4	13,32	212,3	205,7	214,4	111,8	395,5	419,3	218,8	1,934	1,818	9,483
5	16,47	319,5	313,3	321,6	167,8	503,2	528,8	275,9	1,947	1,851	9,658
6	19,62	448,7	444,0	450,8	235,2	633,6	659,6	344,2	1,960	1,885	9,837

$$\lambda^2_{n\,\mathrm{th}} = \lambda_0^2 + \frac{a_n^2 G^2 g_m}{f_m} \equiv \lambda_0^2 + \lambda^2_{nf}, \tag{11}$$

in which $\lambda_0^2 = \gamma_T^2$ and a_n^2 are the roots of the transcendental equation $J_1(a_n) = 0$ [4].

The [5] method was used in solving (6); we speciffied λ_n^2 from (11) and the values for the eigenfunction in the first $\psi_{n0} \neq 0$, with the values for the eigenfuctions in the subsequent steps given by the recurrent formula

$$\psi_{nk} = 2\psi_{nk-1} \frac{1 + \frac{h^2}{2}(\gamma_T^2 - \lambda_n^2 f_{k-1})}{1 + \frac{h}{2}B_{k-1}} - \psi_{nk-2} \frac{1 - \frac{h}{2}B_{k-1}}{1 + \frac{h}{2}B_{k-1}}, \quad N \geqslant k \geqslant 2, \tag{12}$$

in which $B_k = (F'/F)_{\xi = kh}$.

We refined the eigenvalue at which the eigenfuction satisfies the boudnary condition for $\xi = 1$ by division into halves; the start of the search range was derived from the theoretical (11) estimator:

$$l_{n0} = \frac{\lambda^2_{n\,\mathrm{th}}}{10}.$$

Here the $\psi_n(\xi)$ are unsymmetrical functions with respect to the central line of the annular channel and the maximum-velocity line; the normalization condition was taken such that the ψ_n varied in the range [0, 1]: for each λ_n^2, the value of $\psi^*_{n\,\mathrm{nu}}(\xi)$ was scaled to the maximum value of $\psi^*_{n\,\mathrm{nu}}(\xi)$.

The λ_n^2 (n = 0-10), the eigenfunctions $\psi_n(\xi)$, $\bar{\psi}_n$, $\psi_n(0)$, $\psi_n(1)$, and the expansion coefficients C_n were tabulated for the ranges $\mathrm{Re} = 10^4 - 10^6$, $k = 0.1 - 0.9$; $\gamma = 0 - 5 \cdot 10^3$ (with Pr_f = 0.7; Pr_T = 0.8).

Table 1 compares the $\lambda^2_{n\,\mathrm{th}}$ from (11) with the numerical results; the radial coordinate ξ replacing R is accordance with [6] not only extends the wall region and thus increases the integration step in the numerical solution but also enables one to obtain results for high Re and γ because of of the use of the scaled eigenvalues $\lambda_n^2 = (\varepsilon_n G)^2$ and the generalized disequilibrium parameter $\gamma_T^2 = (\gamma G)^2$. The scale parameter G was calculated for the entire range in Re and k from the approximating formula

$$G = 24\mathrm{Re}^{-0.4493} = 0.3828 Y^{-0.4493}\ \delta_{ap} \leqslant 1.6\%. \tag{13}$$

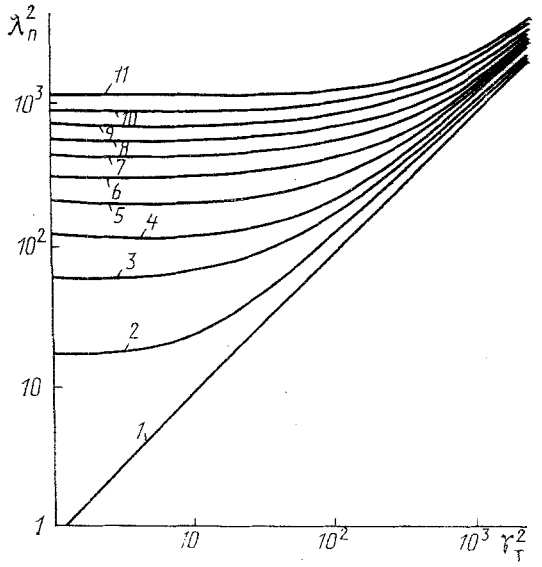

Fig. 1. Reduced eigenvalues λ_n^2 (Re = 10^4-10^6; k = 0.3-0.9; γ = 5-5·10^3): 1) n = 0; 2) n = 1, ..., 11) n = 10.

Figure 1 shows the theoretical and numerical results for λ_n^2 (n=0—10) for all the Re and k; Fig. 2 gives an example of how the eigenfuctions vary across the channel, $\psi_n(\xi)$, for γ = 50 (developed reaction).

Table 1 also gives the (6) eigenvalues λ_{nf}^2 for the Sturm-Luouville problem for $\gamma_T^2 = 0$ (chemcially inert flow).

The heat transport in turbulent flow of a chemically inert material is [1]:

$$\Theta_f = \Theta_{f\infty} + \Theta_{f0}, \quad \Theta_{f\infty} = \bar{\Theta}_f + Gh_f(\xi), \quad \Theta_{f0} = GP_{f0}(\xi, \eta). \tag{14}$$

We represent the solution on the initial thermal section as

$$P_{f0}(\xi, \eta) = \sum_{n=1}^{\infty} C_{nf}\psi_{nf}(\xi)\exp(-\varepsilon_{nf}^2\eta),$$

$$C_{nf} = \frac{-\int_0^1 h_f(\xi) Ff\psi_{nf}d\xi}{\int_0^1 \psi_{nf}^2 Ffd\xi}, \tag{15}$$

in which $\psi_{nf}(\xi)$ is the solution to (6) for $\gamma_T^2=0$ and $h_f(\xi)$ is the solution to the corresponding boundary -value problem [1]. The the heat transfer characteristics in a turbulent inert flow are

$$\Theta_f(0) = \bar{\Theta}_f + Gh_f(0)\left[1 + \sum_{n=1}^{\infty} A_{nf}\exp(-\varepsilon_{nf}^2\eta)\right]; \quad A_{nf} = \frac{C_{nf}\psi_{nf}(0)}{h_f(0)};$$

$$\Theta_{fад}^* = \frac{\Theta_f(1) - \bar{\Theta}_f}{2} = \frac{Gh_f(1)}{2}\left[1 + \sum_{n=1}^{\infty} B_{nf}\exp(-\varepsilon_{nf}^2\eta)\right];$$

$$B_{nf} = \frac{C_{nf}\psi_{nf}(1)}{h_f(1)}; \quad \mathrm{Nu}_{f1} = \frac{2}{\Theta_f(0) - \bar{\Theta}_f}; \tag{16}$$

$$\mathrm{Nu}_{f1}/\mathrm{Nu}_{f1\infty} = \frac{1}{1 + \sum_{n=1}^{\infty} A_{nf}\exp(-\varepsilon_{nf}^2\eta)}.$$

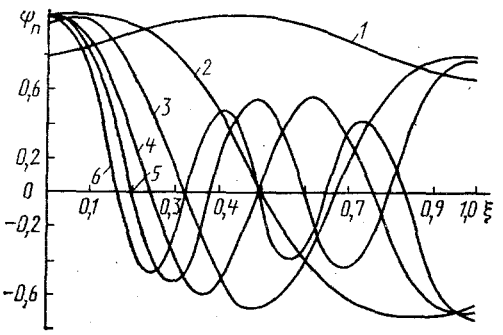

Fig. 2. Eigenfunctions $\psi_n(\xi)$ (Re = 10^5, γ = 50; k = 0.5): 1) n = 0; 2) n = 1; ...; 6) n = 5.

One can compare the results for the Sturm-Liouville treatment for an inert flow derived here and in [7]; the differences over the internal parameters (ε^2_{nf}, C_{nf}, ψ_{nf}) are due to the use of different normalization conditions and solution methods and also the use of different turbulent characteristics ν_T/ν, Pr_T, but they do not lead to discrepancies in the relative Nusselt numbers $Nu_{f1}/Nu_{f1\infty}$ more than 1.3% (Table 2).

We found that $Nu_{f1}/Nu_{f1\infty}$ is dependent not only on z/d_{ec} and k, as [8] implies, but also on Re, which confirms our numerical results on heat transfer.

The theoretical results give the approximating expression ($\delta_{ap} \leqslant 1\%$):

$$\frac{Nu_{f1}}{Nu_{f1\infty}} = F_1(Re,\ k) + F_2(Re,\ k)\left(\frac{z}{d_e}\right)^m, \quad 1 \leqslant \left(\frac{z}{d_e}\right) \leqslant \left(\frac{z}{d_e}\right)_{f\infty}, \tag{17}$$

in which Y = (Re/10^4); m = −0.0906−0.301$Y^{-0.362}$; $F_1 = a_0 + a_1k + a_2k^2$; a_0 = 0.81778−0.24589 log Y + 0.08711 (logY)^2; $a_1 = -0.11796 - 5.422 \cdot 10^{-4} Y^{1.534}$; a_2 = 0.07635 + 0.001149$Y^{1.336}$; $F_2 = b_0 + b_1k + b_2k^2$; b_0 = 0.7009 + 0.10743 log Y − 0.05858 (log Y)^2; b_1 = 0.9665−0.705 log Y + 0.4 (log Y)^2; b_2 = −0.68875 + 0.4825 log Y − 0.30375 (log Y)^2; k = 0.3−0.9; Re = $10^4 \cdot 10^6$.

The length of the initial thermal section in an inert flow is dependent on Re, the channel geometry via k, and the divation $\delta_{f\infty}$ in the Nusselt number from $Nu_{f1\infty}$:

$$\left(\frac{z}{d_e}\right)_{f\infty} \geqslant [2 - 0.25(\delta_{f\infty} - 1)][(28.3 + 46.7k - 33k^2) + (-14.1 - 9.2k + 5.5k^2) Y^{-0,243}],$$

in which $\delta_{f\infty}$ is in percent.

An approximate expression is derived for the change in the relative adiabatic temperature of the outer surface on the initial thermal section:

$$\Theta^*_{fad}/\Theta^*_{fad\infty} = \text{th}\left[(0.0211 + 0.06103Y^{-1} - 0.04063Y^{-2})\frac{z}{d_e}\right]. \tag{18}$$

If one defines $(z/d_e)_{fad\infty}$ as the length at which Θ^*_{fad} differs from $\Theta^*_{fad\infty}$ by not more than 5%, then

$$(z/d_e)_{fad\infty} \simeq 50 + 11(Y - 1)^{0,25}. \tag{19}$$

Generalizing functions have been introduced [1] for convenience in calculating the heat and mass transfer coefficients, which for the initial thermal section are written as

$$\overline{\Phi} = \frac{G\overline{P}}{\overline{\Theta}_f} = \Phi_\infty\left[1 + \frac{\overline{P}_0}{\overline{P}_\infty}\right] \equiv \Phi_\infty\left[1 + \sum_{n=0}^{\infty} M_n \exp(-\varepsilon_n^2\eta)\right],$$

TABLE 2. Comparison of Results for $Nu_{f1}/Nu_{f1\infty}$

Source	z/d_e								
	1	2	3	5	7	10	14	20	30
	Re=2,036·10⁴, k=0,3472								
Procedure (16)	1,659	1,470	1,379	1,281	1,223	1,168	1,122	1,079	1,041
Procedure (17)	1,651	1,461	1,369	1,268	1,210	1,156	1,110	1,017	1,023
[7]	1,629	1,464	1,377	1,281	1,225	1,171	1,125	1,082	1,043
[8]	1,507	1,351	1,277	1,200	1,157	1,118	1,085	1,055	1,026
	Re=1,223·10⁵; k=0,3472								
Procedure (16)	1,534	1,405	1,339	1,263	1,217	1,117	1,113	1,092	1,054
Procedure (17)	1,536	1,409	1,343	1,267	1,222	1,177	1,138	1,100	1,059
[7]	1,545	1,422	1,352	1,270	1,221	1,174	1,133	1,093	1,055

$$S = S_\infty \left(\frac{Nu_{f1}}{Nu_{f1\infty}}\right)\left[1 + \frac{P_0(0) - \bar{P}_0}{P_\infty(0) - \bar{P}_\infty}\right] \equiv S_\infty \frac{1 + \sum_{n=0}^{\infty} N_n \exp(-\varepsilon_n^2\eta)}{1 + \sum_{n=1}^{\infty} A_{nf} \exp(-\varepsilon_{nf}^2\eta)}, \quad (20)$$

$$S_{ad} = S_{ad\infty}\left[1 + \frac{P_0(1) - \bar{P}_0}{P_\infty(1) - \bar{P}_\infty}\right]\left[1 + \frac{GP_{f0}(1)}{2\Theta^*_{fad\infty}}\right]^{-1} \equiv S_{ad\infty} \frac{1 + \sum_{n=0}^{\infty} D_n \exp(-\varepsilon_n^2\eta)}{1 + \sum_{n=1}^{\infty} B_{nf} \exp(-\varepsilon_{nf}^2\eta)},$$

$$\Phi(0) = \frac{GP(0)}{\Theta_f(0)} = \Phi_\infty(0)\left[1 + \frac{P_0(0)}{P_\infty(0)}\right]\left[1 + \frac{GP_{f0}(0)}{\Theta_{f\infty}(0)}\right]^{-1},$$

$$\Phi(1) = \frac{GP(1)}{\Theta_f(1)} = \Phi_\infty(1)\left[1 + \frac{P_0(1)}{P_\infty(1)}\right]\left[1 + \frac{GP_{f0}(1)}{\Theta_{f\infty}(1)}\right]^{-1},$$

in which

$$M_n = \frac{C_n\bar{\psi}_n}{\bar{P}_\infty}; \quad N_n = \frac{C_n[\psi_n(0) - \bar{\psi}_n]}{P_\infty(0) - \bar{P}_\infty}; \quad D_n = \frac{C_n[\psi_n(1) - \bar{\psi}_n]}{P_\infty(1) - \bar{P}_\infty};$$

with $P_0(\xi, \eta)$ defined from (5) and $P_{f0}(\xi, \eta)$ from (15).

We write those functions for the stabilized heat-transfer part [1] as

$$\bar{\Phi}_\infty = \frac{G\bar{P}_\infty}{\bar{\Theta}_f}, \quad \bar{\Theta}_f = 2\eta\frac{k}{1+k}, \quad \Phi_\infty(0) = \frac{GP_\infty(0)}{\Theta_{f\infty}(0)};$$

$$\Theta_{f\infty}(0) = \bar{\Theta}_f + Gh_f(0), \quad S_\infty = \frac{GNu_{f1\infty}}{2}[P_\infty(0) - \bar{P}_\infty], \quad Nu_{f1\infty} = \frac{2}{Gh_f(0)}, \quad (21)$$

$$S_{ad\infty} = \frac{G[P_\infty(1) - \bar{P}_\infty]}{2\Theta^*_{fad\infty}}, \quad \Theta^*_{fad\infty}\frac{Gh_f(1)}{2}.$$

From (20) and (21), we get theoretical formulas for the transfer characteristics on the initial thermal section:

1) mean-mass temperature and composition

$$\bar{\Theta} = \frac{\bar{\Theta}_f}{1+\varkappa}(1+\varkappa\bar{\Phi}), \quad \bar{Y}^* = \frac{\bar{\Theta}_f\varkappa}{1+\varkappa}(1-\bar{\Phi});$$

2) temperature and composition at the inner (heated) surface:

$$\Theta(0) = \frac{\Theta_f(0)}{1+\varkappa}(1+\varkappa\Phi(0)), \quad Y^*(0) = \frac{\Theta_f(0)\varkappa}{1+\varkappa}(1-\Phi(0)); \tag{23}$$

3) temperature and composition at the outer (adiabatic) surface

$$\Theta(1) = \frac{\Theta_f(1)}{1+\varkappa}(1+\varkappa\Phi(1)), \quad Y^*(1) = \frac{\Theta_f(1)\varkappa}{1+\varkappa}(1-\Phi(1)); \tag{24}$$

4) Nusselt number for the inner surface and dimensionless adiabatic temperature for the outer one:

$$\mathrm{Nu}_1 = \frac{(1+\varkappa)\,\mathrm{Nu}_{f1}}{1+\varkappa S}, \quad \Theta^*_{\mathrm{ad}} = \frac{\Theta^*_{f\mathrm{ад}}}{1+\varkappa}(1+\varkappa S_{\mathrm{ad}}). \tag{25}$$

A temperature profile self-similar with respect to the length $\Theta_\infty(0) - \bar{\Theta}_\infty \neq f(\eta)$, $\mathrm{Nu}_{f1\infty} \neq f(\eta)$. is established for certain lengths $(z/d_e)_\infty$ for a turbulent chemically nonequilibrium flow in a ring channel having q_c = constant, as for the flow of a chemically inert medium.

If one define $(z/d_e)_\infty$ as the distance from the inlet at which Nu differs from the stabilized value by not more than $\delta\%$, the length of the initial thermal section in a turbulent nonequilibrium flow in an annular channel is defined by

$$\eta_{\infty\mathrm{Nu}} = 0.1466\,[0.21 - 0.0525\,(\delta-1) + 0.005\,(\delta-1)^2]\,Y^{-0,8986}. \tag{26}$$

The effects from the initial section on the mean-mass characteristics vanish at much shorter lengths:

$$\gamma^2\eta_{\infty\bar{\theta}} \geqslant 4.6 - 0.7635\,(\delta-1)^{0,4597}. \tag{27}$$

At lengths greater than those defined by (26) and (27), one can use approximate theoretical expressions for the generalized functions [1].

The (20) functions vary over the range [0, 1]; when they are 1, chemcial enthalpy exchange in the flow is completely inhibited, and the flow behaves as inert (chemically frozen flow):

$$\bar{\Theta} = \bar{\Theta}_f, \quad \bar{Y}^* = Y^*(0) = Y^*(1) = 0, \quad \mathrm{Nu}_1 = \mathrm{Nu}_{f1}. \tag{28}$$

At the other limit, where they are 0, local thermochemical equilibrium is established:

$$\bar{\Theta} = \frac{\bar{\Theta}_f}{1+\varkappa} \equiv \bar{\Theta}_e, \quad \Theta(0) = \frac{\Theta_f(0)}{1+\varkappa} \equiv \Theta_e(0), \quad Y^* = \varkappa\bar{\Theta}_e \equiv \bar{Y}^*_e,$$
$$\mathrm{Nu}_1 = (1+\varkappa)\,\mathrm{Nu}_{f1} \equiv \mathrm{Nu}_{1e}, \quad \Theta^*_{\mathrm{ad}} = \Theta^*_{f\mathrm{ad}}/(1+\varkappa) \equiv \Theta^*_{\mathrm{ad}e}. \tag{29}$$

These two states are attained asymptotically. One can speak of quasifrozen and quasiequilibrium states. Firstly, the exact definition is dependent on the assumption made about the deviation from the asymptotic states and secondly, (22)-(25) imply that there is an exact description for each heat or mass transfer characteristic.

We define the quasiequilibrium state from the characteristic thermal conditions:

$$\frac{\bar{\Theta}-\bar{\Theta}_e}{\bar{\Theta}_e} \leqslant \delta_{e\Theta}, \quad \frac{\mathrm{Nu}_{1e}-\mathrm{Nu}_1}{\mathrm{Nu}_{1e}} \leqslant \delta_{e\mathrm{Nu}}, \quad \frac{\Theta^*_{\mathrm{ad}}-\Theta^*_{\mathrm{ad}e}}{\Theta^*_{\mathrm{ad}e}} \leqslant \delta_{e\mathrm{ад}}. \tag{30}$$

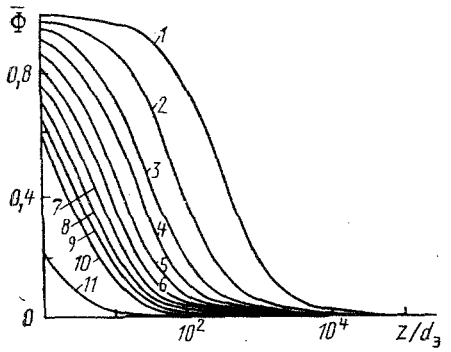

Fig. 3. Generalizing function Φ (Re = 10^5, k = 0.5); 1) γ = 10; 2) 20; 3) 30; 4) 40; 5) 50; 6) 60; 7) 70; 8) 80; 9) 90; 10) 100; γ = 200.

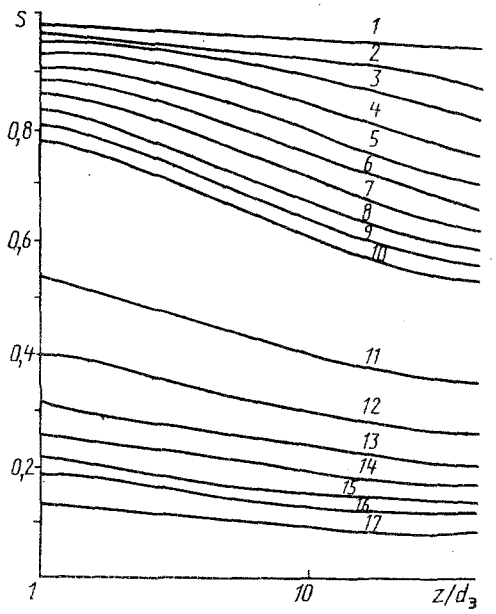

Fig. 4. Generalizing function S (Re = 10^5, k = 0.5); 1) γ = 10; 2) 20; 3) 30; 4) 40; 5) 50; 6) 60; 7) 70; 8) 80; 9) 90; 10) 100; 11) 200; 12) 300; 13) 400; 14) 500; 15) 600; 16) 700; 17) γ = 10^3.

If we take $\delta_e = 5\%$, the criterion for the quasiequlibrium state for all thermal characteristics is put as

$$\gamma \geqslant 7.66 Y^{-0,9227}. \tag{31}$$

The thermochemcial equilibrium state as regards the mean mass characteristics is attained with less stringent constraints on the flow parameters:

$$(\gamma^2\eta)_{e\Theta} \geqslant 40 - 1.565\,(\gamma_T - 1)^n;$$
$$n = 0,529 + 0,1155 \lg Y + 0,0675\,(\lg Y)^2. \tag{32}$$

The quasifrozen state is defined by

$$\frac{\bar{\Theta}_f - \bar{\Theta}}{\bar{\Theta}_f} \leqslant \delta_{f\Theta};\quad \frac{\mathrm{Nu}_1 - \mathrm{Nu}_{f1}}{\mathrm{Nu}_{f1}} \leqslant \delta_{f\mathrm{Nu}};\quad \frac{\Theta^*_{ad} - \Theta^*_{fad}}{\Theta^*_{fad}} \leqslant \delta_{fad}. \tag{33}$$

If we take $\delta_f = 5\%$, the criterion for that state on all the thermal characteristics is

$$\gamma^2\eta_f \leqslant 1.25 - 0.81(\gamma_T - 1)^{0.1842}, \tag{34}$$

and for the mean-mass characteristics

$$(\gamma^2\eta)_{f\Theta} \leqslant 0.107 + 0.0125(\gamma_T - 1). \tag{35}$$

As the (20) functions are four-parameter ones $F_i = F_i(re, k, \gamma, \eta)$ on the initial thermal section, it is not possible to obtain convenient and reliable approximations; for the initial thermal section, one instead has tables and graphs (Figs. 3 and 4).

We have thus derived the theoretical expressions (20) and (21) together with (22)-(25) and these with the [1] approximations enable one to calculate the transfer characteristics on turbulent nonequilibrium flow in an annular channel from the inlet to the part where there is stabilized heat transfer (from the chemically frozen to the equilibrium state). For high temperature differences, the theoretical expressions, the tables, and the graphs should be used with successive approximation as described in [9, 10].

NOTATION

$R = r/\Delta$, $\eta = \dfrac{4z/d_e}{\mathrm{Re}\,\mathrm{Pr}_f}$, dimensionless coordinates ($d_e = 2\Delta$, $\Delta = r_2 - r_1$); $\Theta = (T - T_0)(q_{c1}\Delta/\lambda_f)^{-1}$, dimensionless temperature; $Y^* = (x - x_{0e})\alpha'\left(\dfrac{q_{c1}\Delta}{\lambda_f}\right)^{-1}$, dimensionless NO concentration; $\alpha' = \Delta H_{rM}/C_{pf}\dfrac{M_2}{2}(2 - x_e)$; $\Delta H_{rM} = 2\overset{(3)}{H}_{NO} + \overset{(4)}{H}_{O_2} - 2\overset{(2)}{H}_{NO_2}$, increment in molar enthalpy due to reaction; $\varkappa = \dfrac{\alpha'(x_e - x_{0e})}{(T - T_0)} \simeq \left(\dfrac{\overline{C_{pe}}}{C_{pf}}\right)$, gas reactivity; $\beta^2 = \tau_d/\tau_c$, Damkeller number; $\tau_d = \Delta^2/D_{23}$, characteristic diffusion time; $\tau_c^{-1} = nK_d(2 - x_e)a_1$, chemical relaxation time, n = P/RT, molar density; K_d, dissociation rate constant; x, molar fraction of NO: $\gamma^2 = \beta^2(1 + \varkappa)$, thermal disequilibrium parameter; $f(R) = U_z(R)/\overline{U}$; $g(R) = 1 + \dfrac{\nu_T}{\nu}\dfrac{\mathrm{Pr}_f}{\mathrm{Pr}_T}$; $k = r_1/r_2$; $R_1 = k/(1 - k)$; $R_2 = (1 - k)^{-1}$; $Y = (\mathrm{Re}/10^4)$; $\Theta^*_{ad} = \dfrac{T(1) - \overline{T}}{(q_{c1}\Delta/\lambda_f)}$,

$\xi = \dfrac{1}{G}\displaystyle\int_{R_1}^{R}\frac{1}{\sqrt{g}}\,dR$; $G = \displaystyle\int_{R_1}^{R_2}\frac{1}{\sqrt{g}}\,dR$; $P(0) = P(\xi = 0)$; $P(1) = P(\xi = 1)$; $\gamma_T^2 = (\gamma G)^2$, thermal parameter for chemical disequilibrium in turublent flow; $\lambda_n^2 = (\varepsilon_n G)^2$ reduced eigenvalue; $g_m = g(R = R_m)$; $f_m = f(R = R_m)$. Subscripts: ∞, stabilized heat-transfer range 0, thermal initial section (in P and P_1) and conditions at inlet (T_0 and x_0); t, turbulent parameter; f and e, frozen and equilibrium parameter; M, molar quantity; m, coordiante for maximum rate, approximating error; 1, internal surface.

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